Abdulaziz Al Ghannami
www.ghannami.com

Electrical and Electro-Mechanical Systems

Preliminaries

- Current (i):

$$i = \frac{dQ}{dt} \ or \ Q(t) = \int i \cdot dt$$

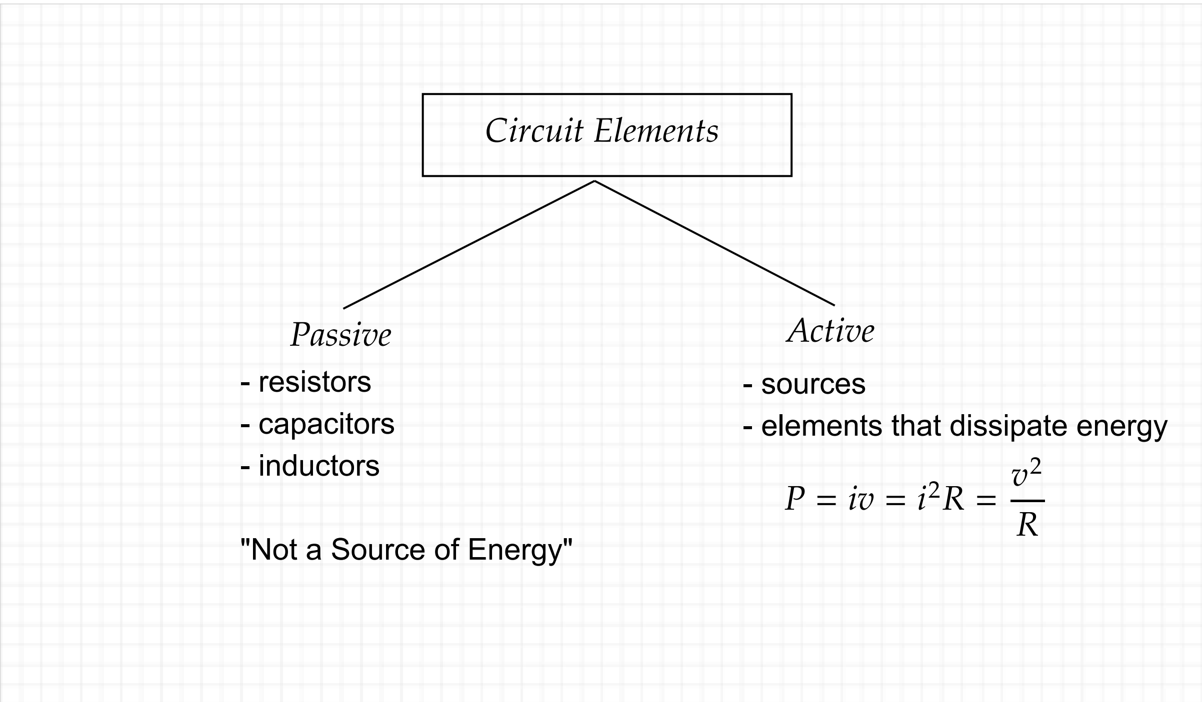
$$i: \ current \ measued \ in \ Amperes \ 'A'$$
$$Q: \ charge \ measured \ in \ Coulombs \ 'C'$$

- Voltage (v): work per unit charge

$$v = iR \ \} \ Ohm's \ law$$
$$i = \frac{v_s}{R},$$

$$where \ v_s \ is \ the \ voltage \ source \ in \ \ measured \ in \ Volts \ 'V'$$

- Circuit Elements



Modeling Circuits

- Conservation of charge $\Leftrightarrow$ Kirchoff's Current Law: Algebraic sum of the voltages around a closed loop or circuit must be zero.
- Sum of currents in a node $=$ sum of currents out of a node
- Series Resistors $\implies R_e = \sum_{i=1}^{n} R_i$
- Parallel Resistors $\implies \frac{1}{R_e} = \sum_{i=1}^{n} \frac{1}{R_i}$

<u>Capacitance (C)</u>

A capacitor is designed to store charge. (PUT SYMBOL)

Capacitance $\implies C \implies$ measures of how much change can be stored for a given voltage difference across the element:

$$v = \frac{1}{C}\int i \cdot dt = \frac{1}{C}\int_0^t i \cdot dt + \frac{Q_0}{C}$$

$$\textit{where } Q_0 \textit{ is the initial charge in a capcitor at } t = 0.$$

$$i = C \cdot \frac{dv}{dt}$$

<u>Inductance (L)</u>

A magnetic field (a flu) surroiunds a moving charfe or current.

Inductor $\implies L \implies$ (PUT SYMBOL) is a passive two-terminal electrical component that stores energy in a magnetic field when an electric current flows through it.

$$\phi = Li \implies \phi = \int v \cdot dt$$

$$i = \frac{1}{L}\int v \cdot dt$$

$$\therefore v = L \cdot \frac{di}{dt} \} \textit{ where } L \textit{ is measured in Henry } (H) \implies (v \cdot s / A)$$

Power & Energy

Power is stored/dissipated by an electrical element is: $P = iv$

E in capacitor: $E = \int P \cdot dt = \int iv \cdot dt = \int \left(C \cdot \frac{dv}{dt}\right) v \cdot dt = c \int vdv = \frac{1}{2}Cv^2$

E in inductor: $E = \int P \cdot dt = \int iv \cdot dt = \int i \left(L \cdot \frac{di}{dt}\right) \cdot dt = L \int idi = \frac{1}{2}Li^2$

P in resistor: $P = Ri^2 = \frac{v^2}{R}$

Electromechanical Systems

They are systems that have electrical and mechanical parts.

$$emf \Longrightarrow v_b = K_b \omega \text{ \} voltage in armature}$$
$$T = K_T i_a \text{ \} torque in armature}$$

$$\text{Usually the case is } K_b = K_T$$

$$\text{where, } v_b \text{ voltage in armature, } K_b \text{ is motion bbacked } emf \text{ \& } K_T \text{ motion torque}$$