KUWAIT UNIVERSITY

Mechanical Engineering Department

ME 421-1 **Spring 2021**

Project (P24) Due Wednesday 16/6/2021

By:

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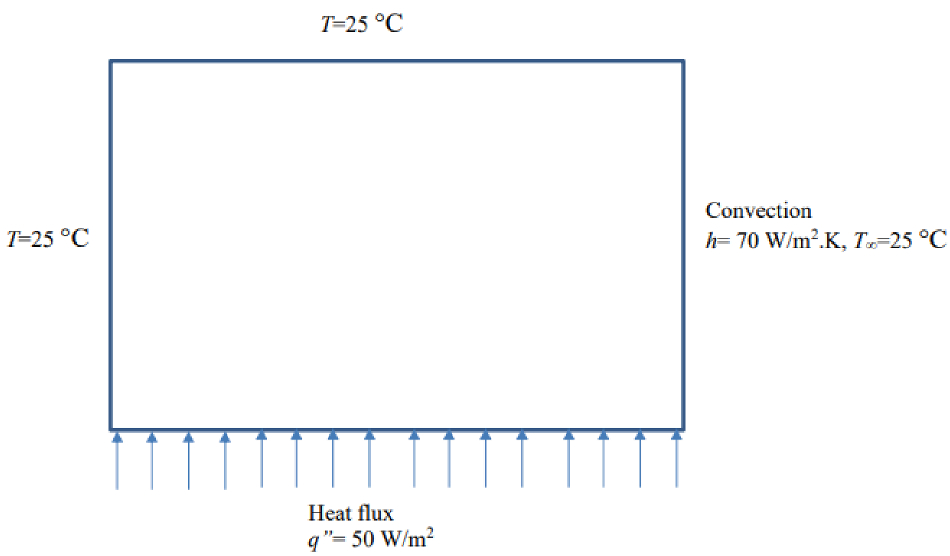
Problem Statement:

Consider steady two-dimensional heat transfer in a long solid body whose cross section is given in the figure. The measured temperatures at selected points of the outer surfaces are as shown. The thermal conductivity of the body is $k = 2$ W/m · °C and there is a uniform heat generation of 3000 W/m^3. The dimension of the solid body is 1.5m ×1m. Using the finite difference method with a mesh size of $\Delta x = \Delta y = 10.0$ cm.

Write a computer code to solve for temperatures of interior nodes.
Determine the temperature of all interior nodes. Present the temperature values in a Table.
Plot a contour plot for the temperatures of the domain



Objectives:

- Create MATLAB code to solve temperatures of interior nodes
- Output a contour plot for the temperatures of the domain
- Output temperature of all interior nodes as a table

Rationale:

The following procedural steps will be followed:

1. System will be represented by a nodal network
2. Obtain a finite-difference equation for the heat equation
3. Define boundary conditions
4. Solve set of algebraic equations for unknown nodal temperatures

Step 1: Grid Generation

Modal network will be represent by a nodal network with mesh sizes of $\Delta x = \Delta y = 10\,cm = 0.1m$.

The system's whole region is, $0 < x \leq 1.5$ & $0 < y \leq 1$.

Dividing into subregions M & N,

$$M = \frac{L}{\Delta x} = \frac{1}{0.1} = 10$$

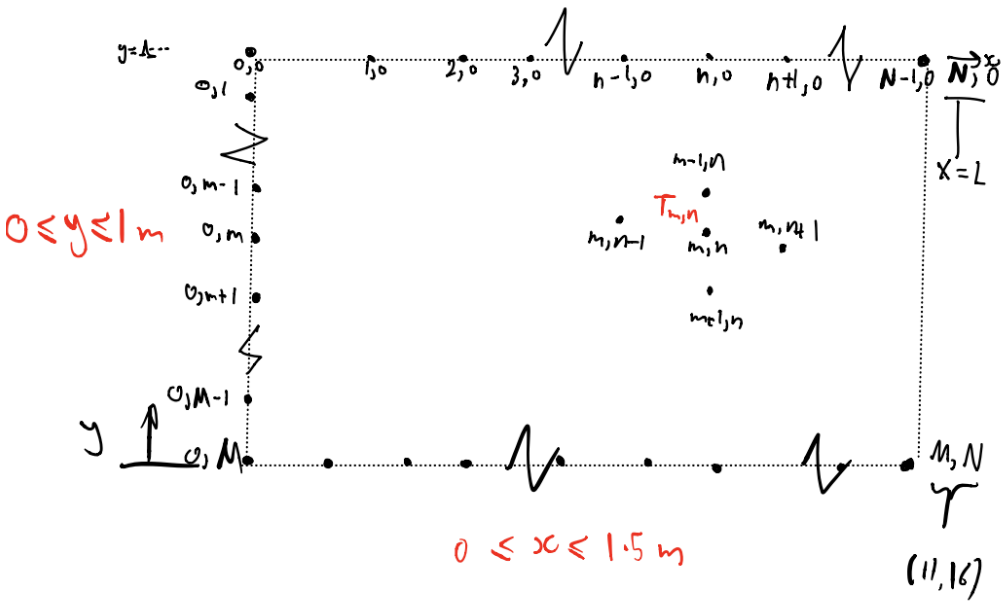
&

$$N = \frac{L}{\Delta x} = \frac{1.5}{0.1} = 15$$

Therefore we have,

$$(M+1)(N+1) = (m)(n) = (11)(16) = 176\ nodes$$

<u>Schematic:</u>



Step 2: Formulate Heat Equation

Assumptions:

- Steady-state
- 2-D convection
- Constant properties
- Heat-flux direction is into the system

Recall that the heat equation is,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{\rho C_p}{k} \cdot \frac{\partial T}{\partial t}$$

Applying assumptions,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\dot{q}}{k} = 0$$

Step 3: Define Boundary Conditions & Interior Nodes

Analysis:

Four Cases of boundary conditions exist on this system:

- Case 1: Prescribed Temperature

- Case 2: Prescribed Heat Flux
- Case 3: Convectional Environment
- Case 4: Interior Nodes

Case 1: Prescribed Temperature:

<u>Location:</u> Left & Top walls

$$T_{Surf} = 25\,^{\circ}C$$

Case 2: Prescribed Heat Flux:

<u>Location:</u> Bottom wall, Bottom-Left corner

Bottom wall:

$$T_{m,n} = \frac{\Delta x}{2k} \cdot q_{m,n} + \frac{\Delta x^2}{4k} \cdot \dot{q} + \left(\frac{2T_{m-1,n} + T_{m,n+1} + 2T_{m,n-1}}{4} \right)$$

Bottom-Left corner:

Case 3: Convectional Environment:

<u>Location:</u> Right wall, Top & bottom right corners

Right wall:

$$T_{m,n} = \frac{\frac{h\Delta x}{k} \cdot T_{\infty} + \frac{\Delta x^2}{2k} \cdot \dot{q} + \left(\frac{2T_{m-1,n} + T_{m,n+1} + 2T_{m,n-1}}{2} \right)}{\left(2 + \frac{h\Delta x}{k} \right)}$$

Bottom-right corners:

$$T_{m,n} = \frac{\frac{h\Delta x}{k} \cdot T_{\infty} + \frac{\Delta x^2}{2k} \cdot \dot{q} + \left(\frac{T_{m-1,n} + 2T_{m,n-1}}{2} \right)}{\left(1 + \frac{h\Delta x}{k} \right)}$$

Case 4: Interior Nodes:

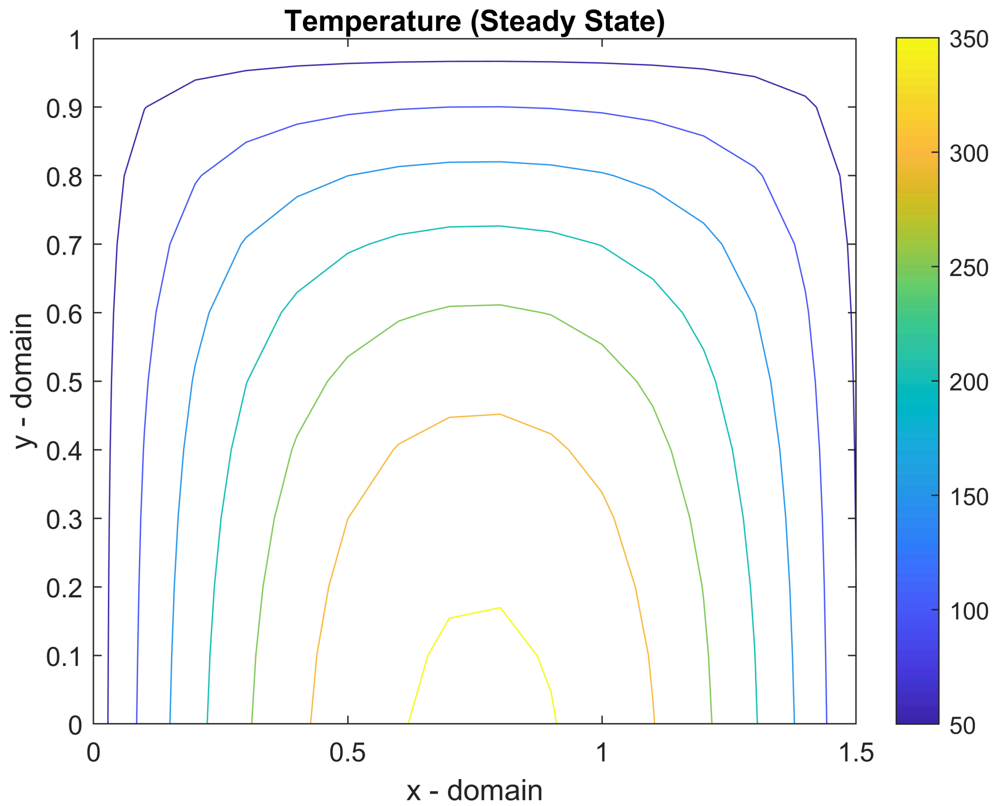
Location: Interior Nodes

$$T_{m,n} = \left(\frac{T_{m+1,n} + T_{m-1,n} + T_{m,n+1} + T_{m,n-1}}{4}\right)$$

Code:

```
% Numerical 2-D Steady Heat Transfer

clear;clc;

% Given

k = 2; % in W/mC
h = 70; % on W/m2
T_surf = 25; % in C
q_gen = 3000; % in W/m3
q_flux = 50; % in W/m2 (Plane with uniform heat flux)
T_inf = 25; % in C
dim = [1.5 1]; % in m
dx = 0.1; % in m
dy = dx; % in m


% Grid Generation (Domain & Step)
m = (dim(2)/dx) + 1;
n = (dim(1)/dy) + 1;



temp = zeros(m,n);
temp(:,1) = T_surf;
temp(1,:) = T_surf;


for iterations = 1:5000

    % Top and Bottom Right Corners, Bottom-Left
    temp(1,end) = 25; % Top-Right corner
    temp(end,end) = (temp(end,end-1) + temp(end-1,end) + q_flux*dx/k + (q_gen/2)...
        *dx^2/2 + T_inf*h*dx/k)/(2 + h*dx/k); % Bottom-Right corner
    temp(end,1) = 25; % Bottom-Left corner


    % Bottom Wall
    for j = 2:n-1
        temp(end,j) = (2*temp(end-1,j) + temp(end,j-1) + temp(end,j+1) + 2 ...
```

```
        *q_flux*dx/k+(q_gen/2)*dx^2)/4; % Bottom wall
    end



    % Right Wall
    for i = 2:(m-1)
        temp(i,end)=(2*temp(i,end-1)+temp(i-1,end)+temp(i+1,end)+2*h*dx...
            *T_inf/k+(q_gen/2)*dx^2)/(2*((h*dx/k)+2)); % Right wall
    end

    % Interior Nodes
    for i=2:(m-1)
        for j= 2:(n-1)
            temp(i,j) = ( temp(i-1,j) + temp(i+1,j) + temp(i,j-1) + temp(i,j+1) ...
                + ((q_gen/2)*dx^2))/4; % interior

        end
    end

end




% Graph for the temperature distribution

rev = temp;

w = linspace(0,dim(1),n);
h = linspace(0,dim(2),m);

 for i=1:m
    for j= 1:n
         temp(i,j) = rev(m-i+1,j);
     end
 end

contour(w,h,temp)
shading interp
title('Temperature (Steady State)')
xlabel('x - domain')
ylabel('y - domain')
c = colorbar;
```




```
% Nodal Network Tabulated
format short
rev = round(rev,3,'significant');
Table = array2table(rev,'VariableNames',{'1','2','3','4','5','6','7','8'...
    ,'9','10','11','12','13','14','15','16'})
```

Table = 11×16 table

...

	1	2	3	4	5	6	7	8
1	25	25.0000	25.0000	25.0000	25.0000	25.0000	25	25
2	25	49.4000	66.3000	78.6000	87.5000	93.8000	98	100
3	25	66.4000	97.3000	120.0000	138.0000	150.0000	158	162
4	25	78.9000	121.0000	153.0000	178.0000	195.0000	207	213
5	25	88.5000	139.0000	179.0000	209.0000	231.0000	246	254
6	25	95.8000	153.0000	199.0000	234.0000	260.0000	277	287
7	25	101.0000	164.0000	215.0000	254.0000	283.0000	302	312
8	25	106.0000	173.0000	227.0000	269.0000	300.0000	321	332
9	25	109.0000	179.0000	235.0000	280.0000	312.0000	334	346
10	25	111.0000	183.0000	241.0000	287.0000	321.0000	343	355

	1	2	3	4	5	6	7	8
11	25	113.0000	186.0000	245.0000	291.0000	325.0000	348	360

```
% n.b. the full Table is not visible via PDF, only visible in the code.
```